Derivative w.s. #2

① a) $f(x) = x^3 - 3x$

Zeros: $x^3 - 3x = 0$

$x(x^2 - 3) = 0$

$x = 0,\ x = \pm\sqrt{3}$

$f'(x) = 3x^2 - 3$

$= 3(x^2 - 1) = 0$

$x = \pm 1$

max/min $x = \pm 1$

NLA

x^2-1	+ ↗		- ↘		+ ↗
		-1		1	

max : $(-1, 2)$

min : $(1, -2)$

$f''(x) = 6x$

$6x = 0$

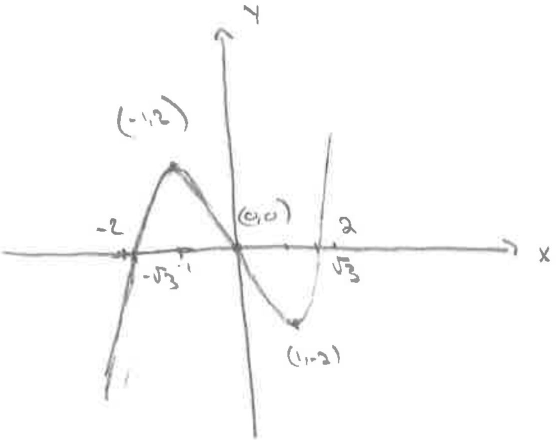
$x = 0$

Point of inflection . $(0,0)$

NLA 6x	-		+
	☹	0	☺

Concave down : $x < 0$

Concave up : $x > 0$



b) $f(x) = 4x^3 - x^4$

Zeros: $x^3(4 - x) = 0$

$x = 0,\ x = 4$

$f'(x) = 12x^2 - 4x^3$

$= 4x^2(3 - x) = 0$

$= x = 0,\ x = 3$

NLA

$4x^2$	+		+		+
$3-x$	+		+		-
	⊕	0	⊕	3	⊖
	↗		↗		↘

max : $(3, 27)$

$f''(x) = 24x - 12x^2$

$= 12x(2 - x)$

$x = 0 \quad x = 2$

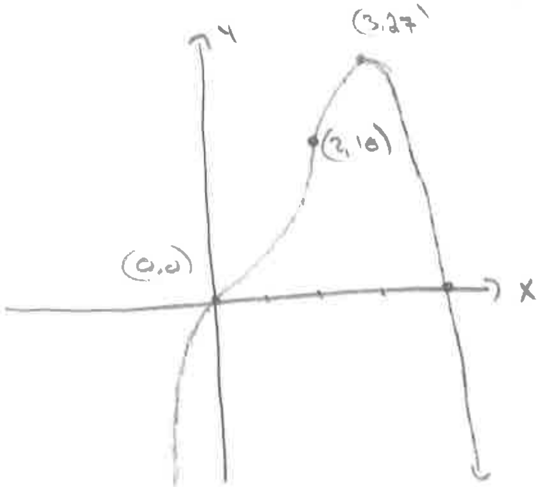
Point of inflection $(0,0)$ $(2,16)$

NLA

$12x$	-		+		+
$2-x$	+		+		-
	⊖	0	⊕	2	⊖
	☹		☺		☹

Concave down : $x < 0,\ x > 2$

Concave up $0 < x < 2$



c) $S(x) = x^3 - 12x + 3$

zeros ?

$S'(x) = 3x^2 - 12$

$= 3(x^2-4) = 0$

$x^2 - 4 = 0$

$x = \pm 2$

x^2-4	+ ↗		− ↘		+ ↗
		-2		2	

Max : (-2, 19)

min : (2, -13)

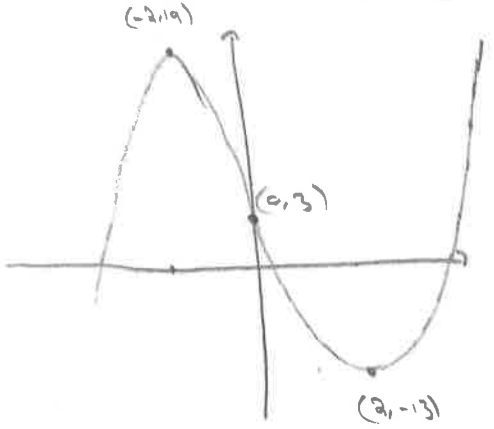
$S''(x) = 6x$

6x	−		+
	⊖	0	⊕

Inflection Point : (0, 3)

concave down : $x < 0$

concave up : $x > 0$



d) $f(x) = 2x^3 - 9x^2 + 12x$

$x(2x^2 - 9x + 12) = 0$

$x = 0, \quad \frac{9 \pm \sqrt{81-96}}{4}$ ⇒ No other real roots!

$f'(x) = 6x^2 - 18x + 12$

$= x^2 - 3x + 2$

$= (x-2)(x-1) = 0$

$x = 2, \quad x = 1$

$x-2$	−		−		+
$x-1$	−		+		+
	⊕	1	⊖	2	⊕
	↗		↘		↗

Max : (1, 5)

min : (2, 4)

$f''(x) = 12x - 18$

$12x - 18 = 0$

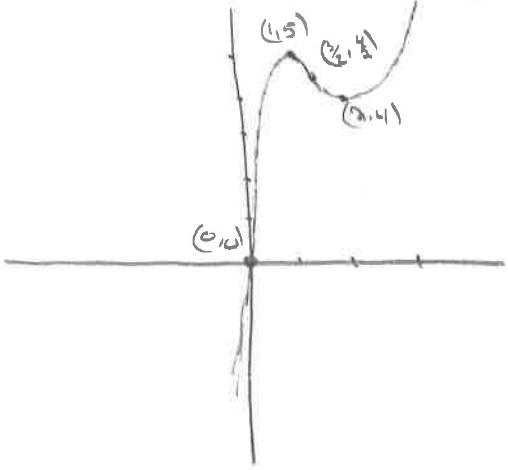
$x = \frac{3}{2}$

$12x-18$	−		+
	☹		☺
		$\frac{3}{2}$	

Inflection Point : $(\frac{3}{2}, \frac{9}{2})$

Concave down : $x < \frac{3}{2}$

Concave up : $x > \frac{3}{2}$



e) $f(x) = 2x^4 - 2x^2$

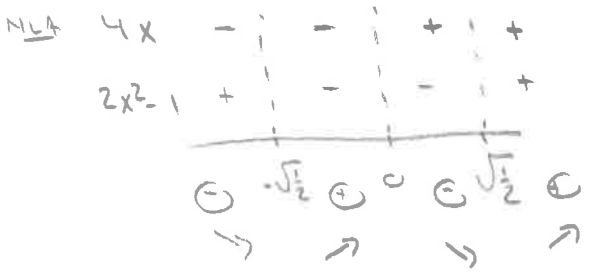
$2x^2(x^2-1)$

$x = 0, \ x = \pm 1$

$f'(x) = 8x^3 - 4x$

$4x(2x^2-1) = 0$

$x = 0, \ x = \pm\sqrt{\frac{1}{2}}$

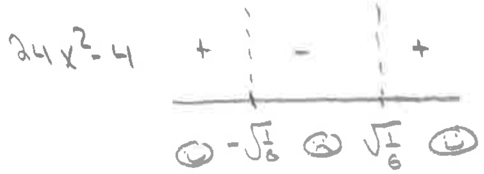


max : (0, 0)

min : $(-\sqrt{\frac{1}{2}}, -\frac{1}{2})$ $(\sqrt{\frac{1}{2}}, -\frac{1}{2})$

$f''(x) = 24x^2 - 4$

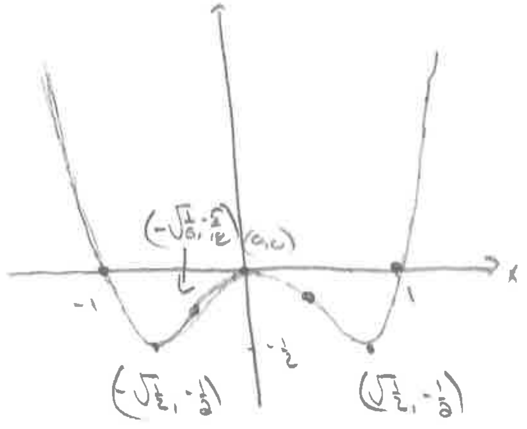
$x = \pm\sqrt{\frac{1}{6}}$



concave up : $x < -\sqrt{\frac{1}{6}}$ $x > \sqrt{\frac{1}{6}}$

concave down : $-\sqrt{\frac{1}{6}} < x < \sqrt{\frac{1}{6}}$

Inflection Points : $(-\sqrt{\frac{1}{6}}, -\frac{5}{18})$ $(\sqrt{\frac{1}{6}}, -\frac{5}{18})$



f) $s(x) = 2x^3 - 3x^2$

$x^2(2x-3)$

Zeros $x = 0, \ x = \frac{3}{2}$

$f'(x) = 6x^2 - 6x$

$6x(x-1) = 0$

$x = 0, \ x = 1$

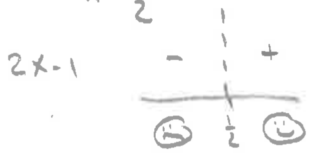
max : (0, 0)

min : (1, −1)

$f''(x) = 12x - 6$

$6(2x-1) = 0$

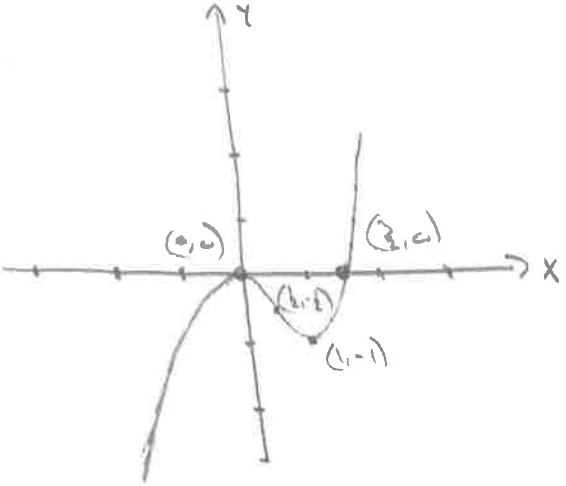
$x = \frac{1}{2}$



Inflection Point : $(\frac{1}{2}, -\frac{1}{2})$

concave down : $x < \frac{1}{2}$

concave up : $x > \frac{1}{2}$



② $f(x) = 2x^3 - 3x^2 - 12x + 20$

a) from calc:

$$\begin{array}{r|rrrr} 2 & 2 & -3 & -12 & 20 \\ & & 4 & 2 & -20 \\ \hline 2 & 2 & 1 & -10 & 0 \\ & & 4 & 6 & \\ \hline & 2 & 5 & 0 & \end{array}$$

$2x + 5 = 0$

$x = -\frac{5}{2}$

Zeros: $x = 2,\ x = 2,\ x = -\frac{5}{2}$

b) $f'(x) = 6x^2 - 6x - 12$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2,\ x = -1$

x-2	-		-		+
x+1	-		+		+
	⊕	-1	⊖	2	⊕
	↗		↘		↗

Increasing: $x < -1,\ x > 2$

c) Normal line ⊥ to tangent

slope tangent $= f'(0) = -12$

$m_\perp = \frac{1}{12}$ Point $(0, 20)$

$y - 20 = \frac{1}{12}(x - 0)$

$y = \frac{1}{12}x + 20$

d) // x-axis ⇒ slope tangent $= 0$

$f'(x) = 0$

$6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1,\ x = 2$

$x = -1,\ y = 2(-1)^3 - 3(-1)^2 - 12(-1) + 20$

$= 27$

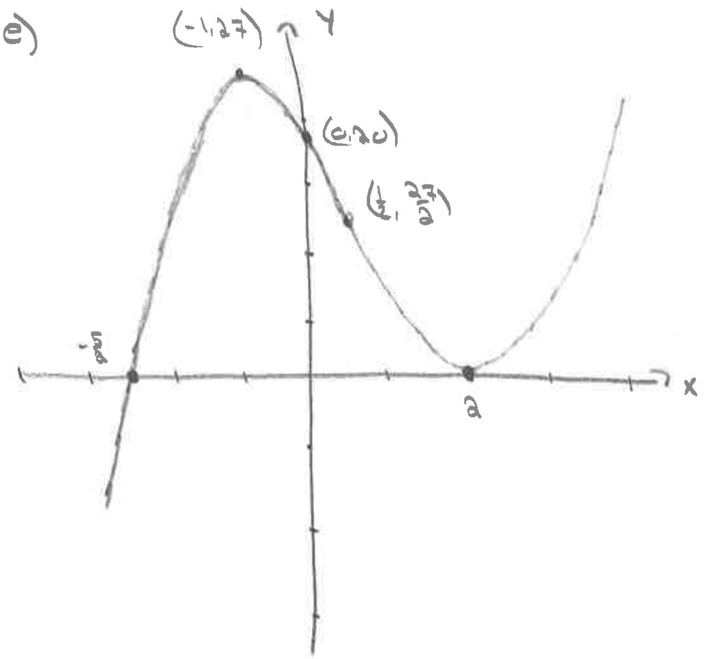
$(-1, 27)$

$x = 2,\ y = 2(2)^3 - 3(2)^2 - 12(2) + 20$

$= 0$

$(2, 0)$

e)



$f''(x) = 12x - 6$

$x = \frac{1}{2}$

$f(\frac{1}{2}) = 2(\frac{1}{2})^3 - 3(\frac{1}{2})^2 - 12(\frac{1}{2}) + 20$

$= 13\frac{1}{2}$

Point of inflection: $(\frac{1}{2}, \frac{27}{2})$

④ $f(x) = x^3 + 3x^2 - 9x + 10$

Zeros: from calc

$X = -5.133$

$f'(x) = 3x^2 + 6x - 9$

$= 3(x^2 + 2x - 3)$

$= 3(x+3)(x-1)$

3	+		+		+
X+3	−		+		+
x−1	−		−		+
	(+)	-3	(−)		(+)
	↗		↘		↗

max: $(-3, f(-3)) = (-3, 37)$

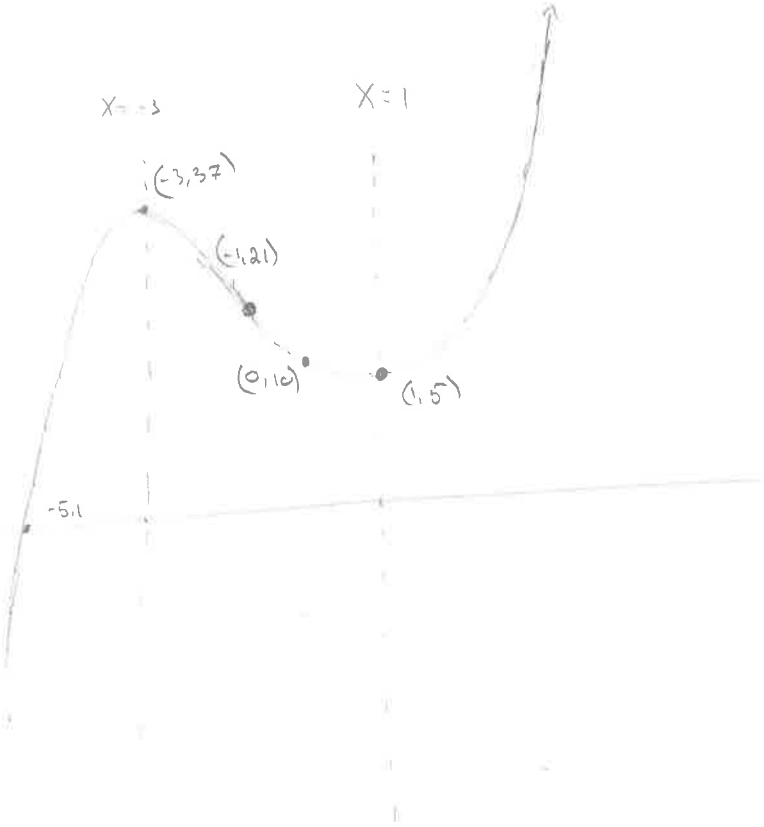
min: $(1, f(1)) = (1, 5)$

$f''(x) = 6x + 6$

$= 6(x+1)$

6	+		+
X+1	−		+
	☹	-1	☺

PoI: $(-1, f(-1)) = (-1, 21)$



⑤ $f(x) = x^3 - 6x^2 + 9x$, $g(x) = 4$

a) $x^3 - 6x^2 + 9x = 4$

from calc: $x = 1$, $x = 4$

b) $x^3 - 6x^2 + 9x = 0$

$x(x^2 - 6x + 9) = 0$

$x(x-3)^2 = 0$

$x = 0$, $x = 3$

c) $f'(x) = 3x^2 - 12x + 9$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3$, $x = 1$

C.I.T.

$f(0) = 0$

$f(1) = 4$

$f(2) = 2$

Range : $0 \le y \le 4$

⑥ $f(x) = x^3 - 9x^2$

a) $x^3 - 9x^2 = 0$

$x^2(x-9) = 0$

$x = 0$, $x = 9$

b) $f'(x) = 3x^2 - 18x$

$3x(x-6) = 0$

$x = 0$, $x = 6$

$3x$	−	+	+
$x-6$	−	−	+

⊕ 0 ⊖ 6 ⊕

↗ ↘ ↗

max: $(0, 0)$

min: $(6, -108)$

increasing: $x < 0$ and $x > 6$

$f''(x) = 6x - 18$

$6x - 18 = 0$

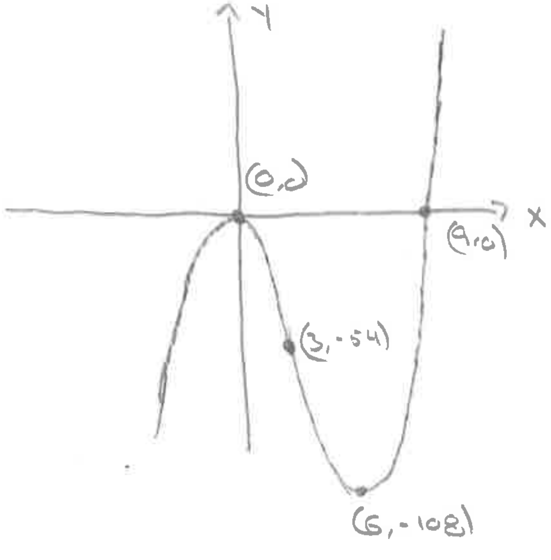
$x = 3$

$6x-18$	−	+

☹ 3 ☺

Point of inflection

$(3, -54)$



③ $g(x) = 2x^3 - 3x^2$

Zeros: $x^2(2x-3) = 0$

$x = 0,\ x = \frac{3}{2}$

$f'(x) = 6x^2 - 6x$

$6x(x-1) = 0$

$x = 0,\ x = 1$

6x	−		+		+
x − 1	−		−		+
	⊕	0	⊖	1	⊕
	↗		↘		↗

Max: (0,0)

min: (1,−1)

$f''(x) = 12x - 6$

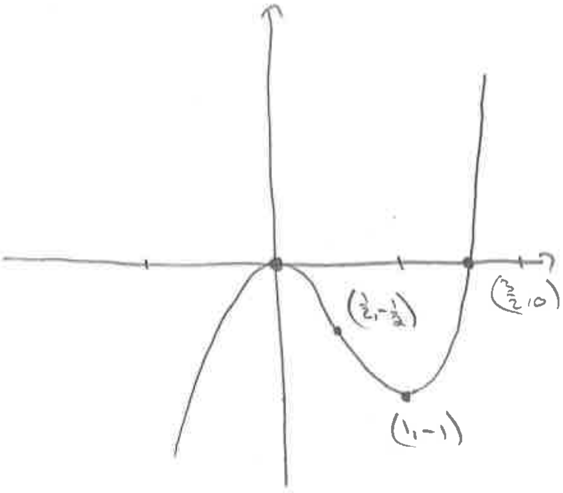
$12x - 6 = 0$

$x = \frac{1}{2}$

12x − 6	−		+
	☹	$\frac{1}{2}$	☺

Point of inflection: $(\frac{1}{2}, -\frac{1}{2})$

$f(\frac{1}{2}) = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$



④ ~~$f(x) = x^3 - 3x^2 - 9x + 10$~~

* Redo used wrong function

~~$f'(x) = 3x^2 - 6x - 9$~~

~~$x^2 - 2x - 3 = 0$~~

~~$(x-3)(x+1) = 0$~~

~~x − 3~~	~~−~~		~~−~~		~~+~~
~~x + 1~~	~~−~~		~~+~~		~~+~~
	~~⊕~~	~~−1~~	~~⊖~~	~~3~~	~~⊕~~
	↗		↘		↗

~~Max: (−1, 15)~~

~~min: (3, −17)~~

~~$f''(x) = 6x - 6$~~

~~$6x - 6 = 0$~~

~~$x = 1$~~

~~6x − 6~~	~~−~~		~~+~~
	~~☹~~		~~☺~~

~~Point of inflection: (1, −1)~~